FORMS OF RELATIVISTIC DYNAMICS: WHAT ARE THE POSSIBILITIES?*

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ABSTRACT

Various methods of constructing solvable few-body models are reviewed, with an emphasis on direct interactions with few degrees of freedom, as an alternative to the use of local quantum field theories. Several applications are discussed.

INTRODUCTION

The subject of relativistic effects in quantum mechanical few-body systems has become a very broad one with an extensive literature. I will attempt to provide an overview of the most commonly used approaches, and describe their distinctive features.

By far the best known approach to this subject is relativistic quantum field theory, with various alternatives based directly upon it. The impressive agreement of the predictions of quantum electrodynamics with experiment, coupled with the realization that a quantized field provided a means for avoiding the concept of instantaneous action at a distance, led to the acceptance of local relativistic field theory as the correct way to model the fundamental interactions of nature at accessible energies.

For the strong interaction, however, *ab initio* calculations based on local field theories are difficult because the infinite number of degrees of freedom and the large coupling constants make it difficult to control the size of the error in any calculation. Field theoretic calculations involve manipulations of a finite number of renormalized Feynman diagrams, using ladder sums or other techniques. These calculations ignore an infinite number of graphs with large coupling constants and they fail to address the extent to which

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the terms in the perturbation series define the dynamics. In addition, most applications in nuclear physics involve composite systems, either of nuclei composed of nucleons, or of nucleons composed of quarks and gluons. The treatment of composite systems in quantum field theories is nonperturbative at the outset. For the case of nucleons as composites of quarks and gluons, the problem is more difficult because the quark and gluon fields do not correspond to observable particles. At present there are no known algorithms for constructing approximate solutions of dynamical problems in strongly interacting quantum field theories with arbitrary precision.

As an alternative to direct solutions to a quantum field theory, one can deal with a set of matrix elements of the field operators, and develop relationships between amplitudes, vertex functions, etc. An approach of this kind is described by Tjon in this session. The alternative considered here is to return to the use of direct interactions, which were considered unacceptable in the search for fundamental theories. Granting the reality that the most successful fundamental theories at hand are indeed local quantum field theories, there remains the question of how to model systems in which the relevant degrees of freedom are not the fundamental ones, *i.e.,* we are working with effective, or truncated, physical systems, such as nucleons in nuclei or quarks in hadrons.

A major advantage of effective interactions is that one can often construct solvable models with a finite, even small, number of degrees of freedom. Typically, eliminating degrees of freedom from a field theory results in nonlocal interactions, and these are usually more easily accomodated within the framework of a direct interaction, which can be non-local, rather than an effective field theory. A notable exception to this is the chiral Lagrangian.

In the remainder of this presentation, I will concentrate on some popular implementations of relativistic direct interactions in quantum mechanical systems. Much of what appears here is presented in greater detail in a recent review article [1].

REQUIREMENTS FOR RELATIVISTIC DYNAMICS

A relativistic quantum mechanical system should consist of operators and state vectors which transform properly under space-time translations, rotations, and Lorentz boosts. In a nonrelativistic system, one substitutes the word "Galilean" for "Lorentz." In this case, all such transformations can be described by means of multiplicative phase factors or simple variable changes in wave functions. The Lorentz case is not as simple. Time translations involve an interacting Hamiltonian in both cases, but Lorentz transformations mix space and time, implying that other transformations must involve interactions as well in order to maintain consistency. Transformations of wave

functions do not necessarily involve simple variable changes. Historically, the insistence upon manifest covariance quickly led to the development of local quantum field theory, which solved this problem at the expense of an infinite number of degrees of freedom. If we give up manifest covariance, then we must be able to show explicitly that states and operators transform consistently. The way to do this is to provide a corresponding set of 10 generators of the Poincaré group: H, $\mathbf{P}$ (space-time translations), $\mathbf{J}$ (rotations), $\mathbf{K}$ (boosts), and demonstrate that these operators satisfy the appropriate commutation relations. Formally, this only needs to be done at the outset, but it is important if covariance is not manifest.

One key commutator is

$$[P_j, K_k] = i\delta_{jk} H \tag{1}$$

Since H is interaction dependent for non-trivial systems, either $\mathbf{P}$, $\mathbf{K}$, or some combination of them must also be interacting. In 1949, Dirac presented three ways of separating interacting and non-interacting generators [2]:

1. **instant form:** $\mathbf{P}$, $\mathbf{J}$ non-interacting, H, $\mathbf{K}$ interacting; the system develops dynamically in time via its associated generator H;
2. **front form:** $P^+ = P^0 + P^3$, $\mathbf{P}_\perp$, K^3, J^3, $\mathbf{E}_\perp = \mathbf{K}_\perp - \mathbf{z} \times \mathbf{J}_\perp$ non-interacting, $P^- = P^0 - P^3$, $\mathbf{J}_\perp$ interacting; the system develops dynamically along the x^+ axis via P^-;
3. **point form:** $\mathbf{K}$, $\mathbf{J}$ non-interacting, H, $\mathbf{P}$; the system develops dynamically along the t axis via H.

In field theories, the Poincaré generators for each of these forms can be constructed from the energy-momentum stress tensor [3].

BAKAMJIAN-THOMAS CONSTRUCTION

In 1953, Bakamjian and Thomas discovered a way to construct a consistent set of 10 generators using direct interactions [4]. The key is to use a set of 10 auxiliary operators $\{\mathbf{P}, \mathbf{j}, \mathbf{X}, M\}$, where $\mathbf{j}$ is the intrinsic spin, $\mathbf{X} = i\nabla_{\mathbf{P}}$, and M is the invariant mass operator. Interactions can be added to the mass operator, while leaving the other nine operators in their non-interacting form. The 10 Poincaré generators are then obtained via

$$H = \sqrt{M^2 + \mathbf{P}^2}; \quad \mathbf{J} = \mathbf{j} + \mathbf{X} \times \mathbf{P}; \quad \mathbf{K} = -\tfrac{1}{2}\{H, \mathbf{X}\} - \frac{\mathbf{P} \times \mathbf{j}}{H + M} \tag{2}$$

Note that H and $\mathbf{K}$ are interacting, while $\mathbf{J}$ and $\mathbf{P}$ are not. This is an instant-form example. If we write $M = M_0 + U$, where M_0 is the non-interacting mass

and U is a potential, then the generators will have the proper commutation relations provided

$$[U, \mathbf{j}] = [U, \mathbf{P}] = [U, \mathbf{X}] = 0. \tag{3}$$

These constraints are precisely those used to restrict potentials in nonrelativistic quantum mechanics: the potential must be a rotational scalar, commute with the total momentum of the system, and be independent of the total momentum.

The eigenvalue equation is

$$M|\Psi\rangle = \lambda|\Psi\rangle. \tag{4}$$

It is useful to consider the related eigenvalue equation

$$M^2|\Psi\rangle = \lambda^2|\Psi\rangle. \tag{5}$$

Instead of writing $M = M_0 + U$, we could instead write $M^2 = M_0^2 + V$, where $V = U^2 + \{M_0, U\}$. Now consider specifically the two-body problem. If $\mathbf{k}$ is the relative momentum between two particles of mass m, the the eigenvalue equation becomes

$$[4(m^2 + \mathbf{k}^2) + V]|\Psi\rangle = \lambda^2|\Psi\rangle. \tag{6}$$

With suitably redefined constants, this is precisely the Schrödinger equation. It implies that phase-shift fits are unchanged if one makes use of potentials which were previously fit using the Schrödinger equation. The binding energy B changes by an amount $B^2/2m$, which is a tiny amount for two nucleons in a deuteron. For meson models represented by a quark and an antiquark, the mass spectrum shifts enough that a new fit would be required.

Note that there are technical similarities to the Schrödinger equation for the two-body problem, but these do not extend to the three-body problem, nor to a system of two particles interacting with an electromagnetic probe.

The Bakamjian-Thomas (BT) idea can also be applied to light-front dynamics. In this case, the 10 auxiliary operators are $\{P^+, \mathbf{P}_\perp, K^3, \mathbf{j}, \mathbf{E}_\perp, M\}$. Once again, interactions are added to the mass operator M, while leaving the other nine non-interacting. The 10 generators $\{P^+, \mathbf{P}_\perp, P^-, J^3, K^3, \mathbf{J}_\perp, \mathbf{E}_\perp\}$ can then be obtained from the auxiliary operators by a similar set of relations to the instant form. The interacting generators depend upon the mass operator as follows:

$$\begin{aligned} P^- &= \frac{M^2 + \mathbf{P}_\perp^2}{P^+}; \\ \mathbf{J}_\perp &= \frac{1}{P^+}\left[\tfrac{1}{2}(P^+ - P^-)(\hat{\mathbf{z}} \times \mathbf{E}_\perp) - (\hat{\mathbf{z}} \times \mathbf{P}_\perp)K^3 + \mathbf{P}_\perp j^3 + M\mathbf{j}_\perp\right]. \end{aligned} \tag{7}$$

The constraint on the choice of mass operator is essentially the same as that in the instant form. One can thus make the same connections to Schrödinger potentials in Bakamjian-Thomas light-front dynamics.

For BT constructions in the point form, the 10 auxiliary operators are $\{\mathbf{V}, \mathbf{j}, \mathbf{Y}, M\}$, where $\mathbf{V}$ is the total velocity operator and $\mathbf{Y} = \nabla_{\mathbf{V}}$. Interactions are added only to M which satisfy the same constraints as in the instant and front forms, and the 10 Poincaré generators are obtained via

$$\begin{aligned} &\mathbf{P} = M\mathbf{V}; \quad H = MG; \quad G = \sqrt{1+\mathbf{V}^2}; \\ &\mathbf{J} = \mathbf{j} + \mathbf{Y} \times \mathbf{V}; \quad \mathbf{K} = -\tfrac{1}{2}\{G, \mathbf{Y}\} - \frac{\mathbf{V} \times \mathbf{j}}{G+1}. \end{aligned} \tag{8}$$

Note in this case that only H and $\mathbf{P}$ depend upon M, as desired.

Beyond Poincaré invariance, another physical requirement of quantum mechanical few-body models is that of cluster separability. By itself, an extension of the Bakamjian-Thomas approach to systems of three [5] or more particles yields an S matrix and/or a set of Poincaré generators which do not cluster properly [6,7]. This difficulty was overcome through the use of packing operators, first introduced by Sokolov [8] and implemented in a different way by Coester and Polyzou [9]. The packing operators are unitary transformations which "pack" the interactions into appropriate subsystems, in a way which restores cluster separability while maintaining the correct commutation relations of the Poincaré group which were established in the BT construction.

APPLICATIONS

The Bakamjian-Thomas approach has been applied to the study of nuclear matter saturation properties [10], a model triton composed of spinless nucleons [11], and electron-deuteron scattering [12]. The model triton problem was solved exactly, while the nuclear matter and deuteron calculations utilized p/m expansions. As discussed below, it is not possible to calculate electromagnetic amplitudes consistently without the introduction of two-body current operators. Thus, a p/m expansion is usually the only option available in actual calculations.

Direct-interaction light-front dynamical models have also been used in a variety of applications.

For systems of three particles, a BT construction can be implemented with two- and three-body forces for any of the three forms of dynamics. Ref. [13]

contains a light-front development. The interacting three-body mass operator is

$$\begin{aligned} M &= \sqrt{M_{12}^2 + \mathbf{K}_{12}^2} + \sqrt{M_{23}^2 + \mathbf{K}_{23}^2} + \sqrt{M_{31}^2 + \mathbf{K}_{31}^2} - 2M_0 \\ &= M_0 + \tilde{V}_{12} + \tilde{V}_{23} + \tilde{V}_{31}, \end{aligned} \tag{9}$$

where $\mathbf{K}_{ij}$ is the three-momentum of the (ij) cluster and

$$\tilde{V}_{ij} \equiv \sqrt{(M_{ij}^{(0)} + V_{ij})^2 + \mathbf{K}_{ij}^2} - \sqrt{M_{ij}^{(0)2} + \mathbf{K}_{ij}^2}. \tag{10}$$

With interactions specified this way, the three-body problem can be formulated in terms of Faddeev equations as well as other methods used in nonrelativistic scattering theory. Ref. [14] provides an application to proton-deuteron scattering in the GeV region, using a multiple-scattering expansion. While p/m is non-negligible in this energy region, it was found that relativistic effects are rather small compared to other effects, such as the sensitivity to the treatment of off-shell intermediate nucleon-nucleon scattering.

The literature abounds with applications of light-front dynamics to electromagnetic processes. There are two sessions of contributed papers in this conference alone which have many such examples. I will summarize here the main motivation for these calculations. For electromagnetic current matrix elements involving spacelike momentum transfer, it is always possible to orient the spatial axes such that $q^+ = 0$. For such a choice of axes, it is sufficient to calculate the matrix elements of $I^+(0)$ in order to determine all observables (*i.e.,* form factors). Furthermore, if one makes the assumption that the calculation is dominated by contributions from one-body currents, these current matrix elements factor out of the matrix element integrals. For example, for spin-$\frac{1}{2}$ constituents, the one-body matrix element is

$$\langle +\tfrac{1}{2}\mathbf{q}_\perp\, p^+; \mu' | I^+(0) | -\tfrac{1}{2}\mathbf{q}_\perp\, p^+; \mu \rangle = \left[F_1(Q^2) - \frac{i}{2m} F_2(Q^2)\, \sigma \cdot \hat{\mathbf{n}} \times \mathbf{q}_\perp \right]_{\mu'\mu}. \tag{11}$$

Two comments are in order:

1. These matrix elements correspond to *on-mass-shell* particles, and as such represent directly measurable quantities (in this case F_1 and F_2. Calculations based upon field theory require input for off-mass-shell current matrix elements even at the one-body level. The difference between these approaches represents a re-arrangement of the dynamics of one- and two-body current matrix elements, thus illustrating the fact that the description of two-body currents is not only not unique, but it also depends upon the choice of two-body interaction.

2. The momentum transfer appearing in this matrix element is the same as the momentum transferred to the composite system. This is not the case for calculations in the instant form, or for light-front calculations where $q^+ \neq 0$, and illustrates the fact that all relevant current matrix elements in the approach described here are related by *non-interacting* light-front boosts.

Calculations of current matrix elements using light-front dynamics allow one to separate the contributions of one-body and two-body currents in a relatively clean fashion. Since the full set of Lorentz transformations and space-time translations necessarily involves interactions at one point or another, the fact that the current operator transforms as a four-vector implies that it must have two-body components. In the nonrelativistic case, only the continuity requirement forces one to introduce two-body currents. In light-front dynamics, interaction dependent two-body currents enter when one considers rotations about a spatial axis perpendicular to the light front. For elastic scattering from particles with spin $j \geq 1$, a consistency condition, sometimes known as an angular condition, can be derived which tests the violation of rotational covariance which results from calculating matrix elements with one-body matrix elements only. The condition can be derived by noting that all Breit-frame matrix elements with helicity transfer greater than unity should vanish. The relevant dimensionless parameter turns out to be $Q/2M$, where Q is the momentum transfer and M is the mass of the composite particle [15]. For two nucleons in a deuteron or three quarks in a baryon, the violation of rotational covariance is a relatively small effect for momentum transfers in the GeV region, but for a quark-antiquark model of a rho meson, there are large violations even at moderate momentum transfers.

THE POINT FORM AND HEAVY QUARK SYMMETRY

The essence of heavy-quark symmetry is that the dynamics of heavy hadrons is controlled by the motion of the heavy constituent quark (t, b, and maybe c), which moves in straight lines unless acted upon by external fields, dragging the other quarks plus glue along with it [16]. Matrix elements of operators which act only on the heavy quark may depend upon the hadronic structure, but not on the heavy quark mass. It therefore becomes natural to express state vectors in terms of velocities rather than momenta:

$$|[mj]\mathbf{p}\mu\rangle \rightarrow |[mj]\mathbf{v}\mu\rangle. \tag{12}$$

In point form dynamics, since rotations and boosts do not depend upon mass, it becomes natural to express state vectors in terms of velocity rather than momenta. For this reason, it is an attractive framework for studying

the heavy-quark limit within models. A key indicator of the heavy-quark limit is the existence of universal form factors $\xi(v' \cdot v)$ which express matrix elements between hadronic states containing a heavy quark, but which are independent of substitution of one heavy quark with one of a different flavor. In a Bakamjian-Thomas direct-interaction model, one can examine explicitly the dependence of such matrix elements on the heavy-quark mass, and develop an expansion in $1/m_h$. A detailed example is given in Ref. [17]

OTHER DIRECT-INTERACTION SCHEMES

The Bakamjian-Thomas construction described above has seen many applications, but it is certainly not the only approach. To motivate some of the alternatives, it is useful to consider what is known as the *world-line condition.*

The trajectory of each particle represents a collection of spacetime points X_i^μ which should transform as four-vectors between inertial frames. That is, the world lines of each particle should transform covariantly. Under a Lorentz transformation, the time dependent position operator $\mathbf{X}^{(n)}(t)$ for particle n becomes $\mathbf{X}'(t')$ via

$$X_j^{(n)}(t) \to X_j'^{(n)}(t') = U^\dagger(\Lambda) X_j^{(n)}(\sigma) U(\Lambda)\Big|_{\sigma=t'} . \tag{13}$$

Note that the transformation U changes the functional form of the operator $\mathbf{X}^{(n)}$, which must then be evaluated at the transformed time t'. To generate a world-line condition, we now require that X^μ also transform in the *geometrical* sense:

$$X^{(n)\mu} \to X'^{(n)\mu} = \Lambda^\mu{}_\nu X^{(n)\nu}. \tag{14}$$

By manipulating infinitesimals, we can convert these conditions to a commutation relation:

$$X_k^{(n)}(0) \left[X_j^{(n)}(0), H \right] = \left[X_j^{(n)}(0), K_k \right] . \tag{15}$$

This is known as the world-line condition. It represents an *additional* constraint upon a dynamical system which comes from requiring the position operator to have a geometrical interpretation. This constraint was examined in detail by Currie, Jordan and Sudarshan [18]. Their study, summarized in what is know as the *No-Interaction Theorem,* concludes that the combined requirements of the Poincaré algebra and the world-line condition for two particles cannot simultaneously be satisfied unless there is no direct interaction! Specifically, they showed that this combined set of commutation relations is unitarily equivalent to the commutation relations for operators for non-interacting particles.

The dilemma posed by the No-Interaction Theorem can also be expressed in terms of four basic ideas [19]:

1. Unitary representation of Poincaré group via a Hamiltonian;
2. world-line invariance
3. equivalence of physical coordinates and canonical coordinates
4. equations of motion hold for all time.

Following the publication of these papers, several alternative approaches to the world-line condition and the No-Interaction Theorem have emerged.

1. *Retain the idea of directly interacting particles, but give up the world-line condition.* This is the essence of the Bakamjian-Thomas approach. The position operator should not be interpreted as an observable. Note also that a world-line implies a sequence of well-defined events. For composite systems such as nucleons, it is even less clear how to construct a position operator with a classical interpretation, particularly in the interaction region [20].
2. *Replace interacting particles with interacting quantum fields.* In this case, X^μ is no longer a dynamical operator, but rather a set of parameters which form the manifold for measuring the fields, which become the new dynamical operators. The framework can be made manifestly covariant, but particle number is no longer fixed.
3. *Retain the world-line condition, but give up the Hamiltonian framework.* This approach was taken by Currie and Hill [21] in developing a set of classical integro-differential equations of motion.
4. *Increase the number of dynamical operators with a corresponding additional number of constraints.* This fourth option is the basis for so-called *constraint dynamics,* first formulated by Dirac in 1964 [22], and is described briefly in the next section.

OVERVIEW OF CONSTRAINT DYNAMICS

Constraint dynamics can be considered as a relativistic extension of classical mechanics problems with holonomic constraints. Rather than reduce the problem immediately to one with the minimum number of degrees of freedom, the problem is cast with extra degrees of freedom, plus additional constraint equations.

In this approach, new dynamical quantities are introduced in such a way that the particles behave as "free" particles in the sense that they satisfy trivially both the world-line condition and the Poincaré algebra. Dynamics

are introduced into the mass-shell condition for the particles, that is, the particles behave as free particles, but with variable mass. This approach is manifestly covariant and satisfies the world-line condition for a fixed number of particles. On the other hand, the interpretation of the dynamical quantities in terms of physically observables in not as clear, and it also difficult to satisfy the additional requirement of cluster separability.

Following Komar [23], we now consider the relativistic quantum mechanics of two spinless particles. We take as our dynamical degrees of freedom the position and momentum four-vectors

$$x_1^\mu, x_2^\mu, p_1^\mu, p_2^\mu, \tag{16}$$

with the usual free-particle commutation relations

$$[x_a^\mu, x_b^\nu] = 0; \quad [p_a^\mu, p_b^\nu] = 0; \quad [x_a^\mu, p_b^\nu] = i\delta_{ab} g^{\mu\nu}. \tag{17}$$

The generators of the Poincaré group are

$$P^\mu = \sum_{a=1}^{2} p_a^\mu; \quad J^{\mu\nu} = \sum_{a=1}^{2} \left(q_a^\mu p_a^\nu - q_a^\nu p_a^\mu\right). \tag{18}$$

So far, the problem has been set up as if the particles were free. There are also too many dynamical degrees of freedom. These points are resolved by modifying the mass-shell conditions for the particles:

$$\begin{aligned} K_1 &\equiv p_1^2 - m_1^2 - \Phi_1(q,p) = 0; \\ K_2 &\equiv p_2^2 - m_2^2 - \Phi_2(q,p) = 0. \end{aligned} \tag{19}$$

These constraints are to be applied only *after* any other operations are carried out. In addition, we require that

$$[K_1, K_2] = 0. \tag{20}$$

This is known as the Todorov-Komar equation.

The constraint equations reduce the number of dynamical degrees of freedom by two. As in the previous example, the operators K_1 and K_2 can *each* generate an equivalence class of states of the system. If $[K_1, K_2] = 0$, then these equivalence classes must themselves lie in the constrained space. Removing these classes then eliminates two more degrees of freedom, bringing the total to 12, which is the desired number.

For the case of two particles, the condition places severe restrictions on the form of the dynamics.

The fact that the individual particle four-vector positions and momenta obey canonical commutations, plus the fact that the Poincaré generators look like those of free particles, means that the No-Interaction Theorem has essentially been sidestepped. A key observation of Sudarshan and collaborators [24] is that in constraint dynamics, the dynamical development of the system is no longer generated by one of the ten Poincaré generators (such as H or P^-). One can define a "time" which parameterizes the dynamical development by defining

$$\mathcal{H} \equiv \alpha K_1 + \beta K_2, \tag{21}$$

and then writing "equations of motion:"

$$\frac{dx_a^\mu}{d\tau} = i\left[x_a^\mu, \mathcal{H}\right]; \quad \frac{dp_a^\mu}{d\tau} = i\left[p_a^\mu, \mathcal{H}\right]. \tag{22}$$

One can then define a "fixation" as a surface of constant τ, but this surface in general does not correspond to any of Dirac's hypersurfaces used in the Bakamjian-Thomas construction.

The connection between observables and operators is not as clear in constraint dynamics as it is in a Hamiltonian framework. One can *define* an operator $\mathcal{O}$ to be observable if it commutes with the constraints:

$$[\mathcal{O}, K_a] = 0, \quad a = 1, 2. \tag{23}$$

In the absence of interactions, this implies that the momenta p_a^μ are observable, while the coordinates x_a^μ are not, even though the latter satisfy the world-line condition. When interactions are introduced, neither p_a^μ nor x_a^μ is observable.

The requirement of cluster separability within constraint dynamics has been studied by Rohrlich and collaborators. As in the case of the Bakamjian-Thomas construction, they find that this formulation of direct interactions also requires the presence of many-body forces in order to preserve separability. They obtain a set of conditions which the constraints $K_a (a = 1, \ldots, N)$ must satisfy for a separable S matrix. A practical solution has not been found for $N > 3$. A constraint dynamical model of two directly interacting pointlike spin-$\frac{1}{2}$ particles has been developed by Crater and Van Alstine [25]. They concentrate on fits to the quarkonium spectrum. Related work is that of Szczepaniak [26].

OTHER APPROACHES

The approaches described above do not constitute an exhaustive list. Considering only attendees to this conference, there have been several recent

contributions. A framework based upon the dynamics of clusters has been developed by Haberzettl [27]. Fuda [28] and Karmanov [29] have developed covariant methods for understanding light-front dynamics. Klink [30] and Lev [31] have suggested different means of constructing two-body operators within a direct-interaction scheme using the point form.

CONCLUSION

Relativistic quantum mechanical few-body systems can be studied within a variety of theoretical frameworks, including both the traditional methods based upon field theory and direct-interaction approaches with few degrees of freedom. Beyond the choice of framework, the ultimate issue is one of dynamics. Most of these approaches are rich enough in structure to accomodate any sort of dynamical input. However, the interpretation of various ingredients such as off-shell effects, two-body currents, three-body forces, as well as what constitutes a relativistic correction, depend both upon the choice of theoretical framework and upon the dynamical input. The approaches reviewed here represent attractive ways of modeling few-body systems which are faithful to a more fundamental physical description and are efficient in terms of relevant degrees of freedom.

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